

A Priori Seminar 09/02/18

Attending: Giovanni Merlo, Josh Thorpe, Sam Symons, Crispin Wright, Paul Conlan, Indrek Löbus, Xintong Wei, Moritz Baron, Alisa Mandrigin, Peter Sullivan, Giacomo Melis, Jonathan Ichikawa, Carrie Jenkins

Reading: Goldfarb, W.. 'Poincaré Against the Logicians'

Presenting: Peter Sullivan

Jonathan suggested that requiring a theory adhere to logical coherence would suffice for it counting as 'logician' on Peter's characterisation of logicism (p. of handout). Peter clarified that he intended his (rough) characterisation to be minimal, somewhere along a spectrum including Hilbert, Frege (no dependence on experience is epistemically relevant), and Naturalism (no distinction between epistemology and experience). Crispin offered that a minimal characterisation of logicism might be that logic plays a structuring role in theory/inquiry: for example, a broad idea might be that mathematical foundations are broadly Euclidean.

Carrie offered an interpretation of the Poincaré/Frege/logician debate using contemporary terms: Poincaré appears to be pushing the importance of doxastic justification (cf. Goldfarb p. 64) not the psychologism with which he is charged, and as opposed to Frege's propositional justification-style project. We might make a tripartite distinction between theories that put at the centre of their theorizing: (i) psychology, (ii) doxastic justification, or (iii) propositional justification. This distinction could further be mapped onto the discovery/justification dichotomy: psychology comes in at the stage of discovery, and epistemology in justification. A question remains though: is the notion of justification that is contrasted with mere discovery doxastic or propositional?

Carrie further wonders whether Poincaré was arguing that justification was a doxastic matter; and his target is those who take it to be propositional. This seems to be suggested by Peter's reconstruction as well, but evidence is there exactly for thinking that Poincaré was and advocate of the priority of doxastic justification (as opposed to an advocate of some psychologism)? Peter responded that Poincaré argues by exclusion to some middle point between the views represented by logicians and naturalists. His intention is to determine some rational basis for our conviction in the status of mathematical/arithmetical truth. Peter then suggested that Goldfarb's point is that Poincaré is unsuccessful because he appears to misunderstand Frege, and that the distinction between psychologism and doxastic justification collapses.

Giovanni asked if the Fregean position only concerned *a priori* knowledge, or empirical knowledge too. If the concern is only *a priori* knowledge, then there is a sense in which the Fregean position can (safely) ignore the question of mathematical thinking or discovery; it is simply not part of the Fregean concern. Peter gave a tentative answer that the Fregean can include cases of empirical knowledge, as conclusions of arguments following from observational premises, but nothing stronger (e.g. premises about empirical facts) can feature while maintaining any interesting (to the Fregean) distinction between empirical and *a priori*.

Jonathan raised the salience of purely rational *empirical* inquiry: for example, calculating the number of chairs in a room by performing arithmetical functions on their arrangement in, say, a grid. He mooted that many theories take speckled hen cases to be a test; whether they can say something principled about the nature of discerning the number of speckles. He referred to our discussion of Sosa in the self-knowledge seminar (week one), where we discussed what can be said where there is

a determinate number of speckles about which one is uncertain. Two lines of thought are that one lacks a rational faculty (which breaks the link between propositional and doxastic justification, cf. Ichikawa and Jarvis); or that there is an experiential deficiency (which means one cannot 'harvest' the available justification for knowledge of there being some particular number of speckles). Peter offered a response on Frege's behalf, saying that he would not be willing to introduce a notion of doxastic justification. Ichikawa and Jarvis prioritise propositional justification, and then move to doxastic, but he (qua Frege) wanted to make no commitment to a theoretical priority between the two notions, nor give one sole rights to 'epistemic resources.'

Crispin offered a route from doxastic justification to propositional: if one reflects on the doxastic justifiers, then one can make propositional justification an idealisation. But this is open to a Wittgensteinian criticism that the resulting notion of propositional justification no longer bears relation to the language game of justification with which we started. Carrie pointed out that this could challenge Goldfarb's reading of Poincaré by widening the latter's theoretical options. Poincaré's position would no longer so easily be deemed psychologistic, as there is a 'legitimate' Wittgensteinian response in the area.

Giacomo distinguished between the focal questions of Poincaré's interest being those of foundational philosophy of mathematics/arithmetic, and our knowledge of mathematics. He suggests that while it seems that Poincaré is wrong with the respect to the metaphysical question of what the foundations are – why should anything psychological matter there? – things may be different with respect to the epistemology of mathematics. Since knowledge requires an agent who forms beliefs, it is at least *prima facie* plausible that some issues having to do with belief-formation matter in the epistemology of mathematics. There is a sense in which the dismissal of belief formation is too quick, if one is interested in more than entailment between truth-functional structures. Giacomo reiterated the importance of having agents in the epistemic/justificatory picture. Peter pointed out that whether there should be agents is precisely what is in question.

Crispin distinguished between investigations of principles about the arrival at knowledge of arithmetic, and principles about acceptance of the logicist framework. The latter (should) require only mathematical resources. If the logicist position is that no rational thinker can arrive at any arithmetical truth without already being an arithmetical thinker, then this is a very different argument from those Poincaré was objecting to. On this sense of the logicist project, one cannot idealise doxastic justification to arrive at a sufficient notion of propositional justification: it either presupposes arithmetical content or disallows casting the idealisation in epistemically salient terms. Peter pointed out this form of logicism is not necessarily in opposition to Giacomo's point about the importance of agents featuring in epistemological theories.

Carrie queried the metaphysical foundations of Frege's/logicist thought: do they derive conclusions about the nature of mathematical facts from certain foundational beliefs (about mathematics/arithmetic)? That is, is there anything greater than a metaphysical fact about the nature of mathematical propositions? Crispin suggested that the Fregean view is to take questions about the epistemology and grounding/metaphysics of mathematics/arithmetic simultaneously. But Carrie raised that if there is any solid ground between doxastic justification and metaphysical foundations, then that must be where Poincaré's targets occupy. Peter pointed out this brings us back to delineating the scope of Poincaré's objection. To take just Frege as his interlocutor would miss the generality of Poincaré's points. He offered a recapitulation Poincaré's point as a focus on the epistemology without the metaphysics; it is to 'bring back the mind' in questions of foundational philosophy of mathematics/arithmetic. Carrie pointed out that there is a position, for example

occupied by Jonathan, which makes good on epistemological requirements without placing psychologistic requirements on justification.

Crispin notes that we should distinguish two ways in which a proof may be in good standing: i) it may be such that an antecedent agnostic can accept the proof's conclusion; or ii) merely express a valid inference-pattern such that to see the ground for the proof one must already know the conclusion. (i) allows an agent to follow a proof from scratch but for this conception to result in foundational mathematical/arithmetical knowledge the agent must be supplied with additional knowledge to recognise the proof as a part of those foundations.

Peter suggests that Goldfarb's argument (against Poincaré) fails inasmuch as one would not have a proof unless one it was "brought together in one mind," along with a recognition of the grounding of the proof, which is only possible if that mind has the capacities to recognise the relevant factors. Carrie gave an interpretation of 'proof' for those worried about psychologism: a proof only requires there be a relation (between premises and conclusion), to bring in a requirement of appreciation of a proof as a proof is already psychologistic. Peter raised that so long as one questions what a proof is, questions about minds will re-enter the discussion.

Jonathan brought up that recognition of a proof as a proof opens the door for the points made in the Lewis Carroll note. Peter clarified that the task is to identify recognition of the adequacy of grounds for a proof based on no 'psychological' interference. Crispin gave a cursory suggestion that this must be on the basis of nothing whatsoever or some mysterious second option (some philosophical holy grail).