KBNS A Priori Seminar 1 **15/10/18**

[Add Reading Title]

Attending: Indrek, Sonia, Moritz, Crispin, Giovanni, Paul, Sam, Xintong, Eduardo, Peter, Claire, Giacomo, Carrie, Jonathan, Fiona, Philip.

Presenting: Giacomo, Giaquinto, *Visual Thinking in Mathematics*, chapters 1-2.

Giovanni wonders why Giaquinto's characterisation of an 'epistemically acceptable' form of discovery was conjunctive: "(i) discovery requires coming to believe something by one's own light … and (ii) discovery only requires absence of irrationality" (Handout, 1). **Crispin** pointed out this would help avoid cases of clairvoyance and suggested that discovery probably requires more than mere reliability. **Giovanni** suggested that epistemic rationality already subsumes reliability, but **Crispin** pointed out that reliability would have to be a separate matter there too. Brains in Vats can be epistemically rational without being reliable: they can have systems of epistemically rational beliefs without those beliefs 'matching' reality. **Giacomo** suggested there might be some set up here for later introduction of an internal/external distinction.

Philip questioned why and how **Giaquinto** could account for some proofs one can provide do 'better' at explaining the truth of 'p' (that which is proved) than others. He flagged that **Giaquinto** was assuming that proof and explanation can come apart.

Paul asked whether one can understand a proof without also discovering it, which **Giacomo** agreed seems to be a possibility on Giaquinto's view.

Crispin pointed out that Giaquinto assumes geometry is 'Euclidean' [foundationalist], in that it 'flows' from some (maybe) intuitive basic principles. **Carrie** questioned this (default) methodology, and **Crispin** suggested Giaquinto needed to provide a positive reason that geometry was not justified in *medias res* or *ex nihilo*.

Carrie was surprised at Giaquinto's seeming characterisation of mathematical knowledge in what seem to be purely reliabilist terms (Chapter 2, p. 12). **Crispin** added his confusion about the seeming disappearance of Giaquinto's requirement for the absence of irrationality, in particular he wondered how the absence of irrationality might be satisfied as a constraint in basic geometrical knowledge. He suggested it might be something which we have by default.

Peter wondered whether our rules for possessing geometrical concepts could be tonk-ish. His further concern was how we could then go wrong in the application of such concepts and suggested this might obtain when there are inherent problems with the concepts in question. **Giacomo** suggested that, because of their supposed close link with perceptual concepts, basic geometrical concepts may be immune from tonk-like worries, and misapplications due to theoretical misunderstandings.

Crispin asked whether there was anything like an enabling/warranting distinction in Giaquinto's view. **Sam** suggested that Giaquinto's understanding/discovering distinction might be an attempt to cut across the enabling/warranting distinction: understanding involved something like a warranting role for experience, whereas discovery only involved an enabling role. **Giacomo** thought that what Giaquinto calls 'discovery' is really an informal proof; if so, presumably it's something that plays a warranting role.

Jonathan asked what special role is Giaquinto giving to visual perception in the acquisition of geometrical concepts: what is special about vision in particular? Perception is (generally) used in a much broader way than just denoting visual perception. **Carrie** noted the loose use of terms creates a problem for comprehending Giaquinto, who has a tendency to slide between perception and visual perception. This makes it difficult to determine where the epistemic significance of visual thinking lies with respect to the epistemology of mathematics. **Jonathan** referred back to the original Meno (which Giaquinto appears to be reversing in some way) which had Socrates' interlocutor 'remember' mathematical facts rather than having imagination play any (significant epistemic) role. **Giacomo** notes that Meno's slave is led by Socrates to follow some visual reasoning, and agrees with Jonathan that Giaquinto is implicitly assuming that Plato's idea that the slave recollected what he already knew is mistaken.

Claire wondered why Giaquinto focused on the retina in his discussion of (cognitive) science, especially as the book is named for 'visual thinking' and not 'vision'? One could (presumably) acquire the necessary resources for visual thinking from, for example, braille. **Giovanni** pointed out that Giaquinto defined the dispositions to believe that an object is a certain shape/mathematical object in terms of visual perceptions. He then wondered what was the relevant 'we': people with 'normal' eyesight in 'normal' conditions? Once this has been ascertained, we can then ask how do 'our' (for the relevant 'our') perceptual capacities contribute to geometric knowledge?

Crispin asked what is a 'perceptual outcome'? Is it something conceptual. **Giacomo** clarified that it was only conceptual when a subject recognises that outcome as [something], not merely perceives it.

Matt asked, given that the 'category specification' of a shape is not the same as the shape's concept, what is extra in the concept? **Giacomo** clarified that the category specification involves no reference to an agent, whereas the concept does. To possess a concept is, in part, to have certain inferential dipositions.

Carrie pointed out that a large objection to Peacocke style views is Williamson's, especially for such views with concepts being tied to dispositions. The Williamson line is that we can be misled about logic but still be said to possess the concept of, say, the conditional. Giaquinto does not seem to recognise the need to answer this objection.

Paul questioned the basicness of shape concepts. If we already have a basic concept of square, why is Giaquinto's visual concept of square defined in terms of other geometric notions. **Peter** added that even if a visual system does divide up concepts so, a subject does not have to.

Handout

KBNS A PRORI SEMINAR
18/09/2018
GIACOMO MELIS

Marcus Giaquinto, *Visual Thinking in Mathematics*
Chapters 1-2

Chapter 1

Chief aim of the book: “show, how, why, and to what extent” the view according to which visual thinking in mathematics does not play an epistemological role is mistaken. (1)

Empirical dimension of Giaquinto’s investigation: since epistemological questions concern the evaluation of a cognitive state acquired in a certain way, they demand an account of the way in which the relevant cognitive state is acquired. When it comes to mathematical beliefs formed through visual thinking, we need to address the following questions:

(i) what kind of visual representations are used?
(ii) How are they deployed?
(iii) What is the nature of the causal route from visual experience to mathematical belief?

Answering these questions requires engaging with the cognitive science that deals with visual perception and visual imagination. (1-2)

Proof vs discovery: To discover a truth one must come to believe it *independently* and in an *epistemically acceptable* way—i.e. a way that is reliable and involves no violation of epistemic rationality. (2)

Discovering that P is to be distinguished from obtaining a justification to believe P because: (i) discovery requires coming to believe something by one’s own light (hence, contrary to justification, is incompatible with testimony), and (ii) discovery only requires absence of irrationality, while justification (presumably) demands a positive contribution of rationality. (2)

This may be a bit too fast: some people would characterize epistemic justification in terms of epistemically blameless belief, and would think of coming to believe by one’s own light in an epistemically respectable manner as a way of getting justification. If so, perhaps what Giaquinto calls ‘discovery’ is just a species of justification (proof would be another).

Giaquinto may be using “proof” and “justification” as near synonymous. A proof is characterized as “deduction from the axioms” (5). [There is a question as to whether visual thinking may play an epistemic role in proof as well (Giaquinto thinks that it may), but we won’t touch on that. Our focus will be on discovery]

Historical note on philosophers’ interest in mathematical discovery: while philosophers such as Plato, Kant, and Mill where very interested in mathematical discovery, at the end of the 19th century the reliability of visual thinking in analysis came under suspicion and, among other things, led to the attempt to “give mathematics a rigorous reformulation in terms of numbers and classes, without reference to anything spatial”. Philosophers then focused on the justification of whole bodies of mathematics (e.g. number theory, analysis, and set theory), and the question of individual discovery dropped off the picture.

“It was simply assumed that mathematical knowledge would have to be a matter of proof, that is, deduction from the axioms; the only question, then, was how the axioms and inference rules of the relevant axiomatic systems could be justified”. (5)

Eventually, in more recent years, the problems associated with attempts to justify axiomatic systems (the various paradoxes, the unpalatable consequences of holistic empiricism and conventionalism, the mystery of mathematical intuition…) have led to a growing interest in mathematical practice. Giaquinto emphasizes that mathematical practice is more than just the activity of proving theorems; rather, it also includes “discovering truths, explaining them, formulating axioms or definitions, constructing problem-solving techniques, constructing methods for application, and devising symbol-systems”. Moreover, each of these activities involves aspects of *making it*, *presenting it*, and *taking it in*. While makers are typically mathematicians, presenters are often teachers, and even schoolchildren are takers-in. Thus, all of us engage with mathematical activity at some point. This brings the question of individual discovery back to the centre of the stage.

Question: *how is the goal to account for the justification of an axiomatic system related to the goal to account for mathematical discovery? Are they two distinct and unrelated projects? Or is the investigation into mathematical discovery expected to offer insight that may be used in new accounts of the justification of axiomatic systems?*

Giaquinto’s focus is on discovery; more precisely, on discovery that arises from visual thinking.

Chapter 2

<u>Big question</u>: how can we acquire *basic* geometrical knowledge?

Any geometrical discovery relies on some starting points. “Basic” starting points are those that are not the result of prior reasoning.

“In order to be starting points for genuine discovery, it is not enough that these [starting points] strike us as obvious. We must actually know them to be true”. (12)

This may sound as if Giaquinto is setting the bar too high. One could wonder whether a positive epistemic evaluation short of knowledge (i.e. justification or warrant) would be good enough; if so, one might also wonder whether a starting point may be justified by the fact that it strikes us as obvious. But perhaps this is just a terminological worry. Giaquinto’s objection may be rephrased in terms of justification: “why would the fact that some proposition strike us as obvious be enough for justification?”

<u>Outline of the answer</u>: “Our initial geometrical concepts of basic shapes depend on the way we perceive those shapes. In having geometrical concepts for shapes, we have certain belief-forming dispositions. These dispositions can be triggered by experiences of seeing or visual imagining, and when that happens, we acquire geometrical beliefs. The beliefs acquired in this way constitute knowledge, in fact synthetic a priori knowledge, provided that the belief-forming dispositions are reliable”. (12)

Needless to say, we will be especially interested in how exactly a warranted belief can be both triggered by experiences of seeing or imagining <u>and</u> be independent of those experiences with respect to what plays the evidential role.

Aspects of two-dimensional shape perception

<u>Border assignment</u>: Visual perception does not work only by producing representations of the retinal image. Rather, “visually perceiving edges or borders of surfaces does not necessarily involves seeing lines that mark those borders. Perceptual borders can be constructed from contrasts in the luminance of adjacent regions, or from lines, or by more indirect means…”. (13-4) (see figure 2.1)

“When one part of the top of a T junction is common to two regions, the border containing the top of the T is assigned to the region that does not contain the stem of the T (14)…[but] in two-dimensional shape perception, our main concern, common borders are assigned to both regions, regardless of cues that would be used in depth perception.” (15) (see figure 2.2)

Mach effect: the perception of a figure is affected by its orientation (15). See figure 2.3 of the square-diamond.

Reference System (RS): the orientation of a figure is relative to a RS, which may be described as “a pair of orthogonal axes, one of which has assigned an ‘up’ direction. A reference system can be based on features of the perceived object, on the perceiver’s retina, head, or torso, on the edges of a page, or on the environment.” (15-6) A change of reference system may very well change the perceptual outcomes (see figure 2.4)

Perceptual processing prefers some reference system to others: in the absence of additional factors, the visual system prefers environmental axes to retinal axes.

The upshot is that the description of visual representation of figures will have to specify the relevant reference system. The reference system of a square perceived as a diamond will be different from that of a square perceived as a square. (16)

Description sets: Giaquinto will characterize geometrical concepts (partly) in terms of descriptions sets. They are representations of visual features which are encoded in a format that has neural realization (i.e. not a natural language), and perceivers do not normally have conscious access to them. (17)

Perception vs perceptual recognition: “perception involves generating a set of descriptions of what is perceived; recognition involves this and the additional step of finding a best match between the generated description set and a stored description set for the conventional appearance of the figure”. (17)

The move from mere perception to perceptual recognition may be done by selecting a new RS at will. This may lead to discovery: e.g. “all squares are diamonds”. (17-18)

Intrinsic axes: some RS may also be overridden without conscious directed attention, if the relevant figure has a strong intrinsic axis. (see figure 2.5)

Reflection symmetry: The intrinsic axis of a figure as it is perceived by the visual system may be determined by a number of things, including the length of lines that fall inside the figure’s boundaries, and the so called “reflection symmetry”—a symmetry that determines a RS in virtue of the way several elements of the perceived figure are related. (18-9) See figure (2.6) for RS determined by reflection symmetry, and figure 2.7 for an example of reflection symmetry that overrides the Mach phenomenon.

“When a symmetry axis of one figure coincides with the symmetry axis of one or more surrounding figures, the visual system is more likely to use that axis as the main up-down axis for feature descriptions”. (20)

“Symmetry perception involves two processes: a fast but rough test of reflection symmetry in all orientations simultaneously; then, if one or more axes of symmetry are detected, a more precise evaluation of symmetry about one or more of these axes in turn” (21)

The role of reflection symmetries is not limited to individuating the main axis that determine the RS, but they are also involved in individuating further features that contribute to determine what figure is perceived exactly. By considering the squares in figure 2.9, we can see how the symmetry of the square in its normal orientation requires perceiving every pair of adjacent angles as equal (hence all angles as equal), while the symmetry of the diamond requires perceiving all pairs of opposite angles as equal (hence not all angles as equal). On the other hand, the symmetry of the diamond requires that we perceive all sides as equal, whereas the symmetry of the square does not require that we do perceive all sides as equal. Yet, we do perceive all sides of a square as equal (otherwise we would be perceiving a rectangle), and we do so courtesy of a secondary sets of orthogonal axes—axes at 45° of the primary axes. (21-3)

<u>A category specification for squares</u>:

Plane surface region, enclosed by straight edges:
  Edges parallel to H, one above and one below; edges parallel to V, one each side.
Symmetrical about V.
Symmetrical about H.
Symmetrical about each axis bisecting angles of V and H. (23)

The presence of the foregoing features in a perceived figure determine that the perceived figure is a square.

The capacity to reason about squares requires possession of the concept of a square. The concept of a square needs to be distinguished from the perceptual category specification for squares—even if, being <square> a concept of a perceptible kind, it is closely related to its perceptual category specification.

<u>A theory of concepts (based on Peacocke 1992)</u>: a concept is a constituent of a thought; a thought is the content of a possible mental state which may be correct or incorrect (true or false), and has inferential relations with other such contents. Thoughts and concepts may be expressed linguistically, but they are not to be identified with their linguistic expressions. (24-5)

Concepts may be identified in terms of the inferential relations they stand in: "to possess a concept one must be disposed to find certain inferences cogent without supporting reasons. So, we can in principle specify a concept in terms of these basic inferences". Example of the concept <uncle>. (25)

The view outlined admits non-conceptual contents. Since the specification of a concepts appeals to its relation to other concepts, one might wonder how concept-possession may get off the ground in the first place, it would seem that the possession of any concept requires the possession of some stock of concepts in the first place: where does such stock come from? The objection may be bypassed by allowing that concepts may be specified also in terms of transitions from non-conceptual content-bearing states other than from thoughts. (25-6)

Of course, this is a view that has been questioned, but Giaquinto does not make any effort to defend it.

<u>A perceptual concept for squares</u>: (the geometrical concept is a restricted version of it)

The general idea is that "the perceptual concept for squares centres on a disposition to judge something as square when it appears square and one does not suspect that circumstances are illusiogenic or one's vision is malfunctioning". (26)

Something can appear square to one even if one is not deploying (or cannot deploy) the perceptual concept for squares (otherwise the above suggestion would fail to specify the concept). That's because, as we have seen above, one can perceive something as square without recognizing it as square. (26)

The concept **{square}** is the concept C that one possesses iff both of the following hold:

(a) When an item *x* is represented in one's perceptual experience as a n/c plane figure n/c enclosed by n/c straight edges, one edge above H and n/n parallel to it, one below H and n/c parallel to it, and one to each side of V and n/c parallel to it, and as n/c/ symmetrical about V and n/c symmetrical about H—when *x* is thus represented in perceptual experience and one trusts the experience, one believes without reasons that that item *x* has C. Conversely, when one trusts one's perceptual experience of an item *x*, one believes that *x* has C only if *x* is represented in the experience as n/c plane figure n/c enclosed by n/c straight edges…

(b) Let "Σ" name the shape that figures appear to have in the experiences described in clause (a). When an item *x* is unperceived one is disposed to find inferences of the following form cogent without supporting reasons:

x has Σ. Therefore *x* has C.
x has C. Therefore *x* has Σ.

The *geometrical* concept for square is a restricted version of the perceptual content. It centres on the disposition to judge something as square just when it appears *perfectly* square (the "n/c" qualifications are dropped). (29) Something can appear perfectly square in virtue of the limitations of our visual acuity: some imperfections are so slight that cannot be perceived by us. (28)